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Note on the Thermal Behavior of the Neutron Electric Dipole Moment from QCD Sum Rules

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Abstract

We use the method of thermal QCD sum rules to investigate the effects of temperature on the neutron electric dipole moment d_n induced by the vacuum $\bar{\theta}$-angle. Then, we analyze and discuss the thermal behaviour of the ratio $\mid \frac{d_n}{\theta} \mid$ in connection with the restoration of the CP-invariance at finite temperature.

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1 Introduction

The CP symmetry is, without doubt, one of the fundamental symmetries in nature. Its breaking still carries a cloud of mystery in particle physics and cosmology. Indeed, CP symmetry is intimately related to theories of interactions between elementary particles and represents a cornerstone in constructing grand unified and supersymmetric models. It is also necessary to explain the matter-antimatter asymmetry observed in universe.

The first experimental evidence of CP violation was discovered in the $K - \bar{K}$ mixing and kaon decays [1]. According to the CPT theorem, CP violation implies T violation. The latter is tested through the measurement of the neutron electric dipole moment (NEDM)d_n. The upper experimental limit gives confidence that the NEDM can be another manifestation of CP breaking. To investigate the CP violation phenomenon many theoretical models were proposed. In the standard model of electroweak interactions, CP violation is parametrized by a single phase in the Cabbibo Kobayashi Maskawa (CKM) quark mixing matrix [2]. Other models exhibiting a CP violation are given by extensions of the standard model; among them, the minimal supersymmetric standard model $MSSM$ includes in general soft complex parameters which provide new additional sources of CP violation [3, 4].

CP violation can be also investigated in the strong interactions context through QCD framework. In fact, the QCD effective lagrangian contains an additional CP-odd four dimensional operator embedded in the following topological term:

$$L_\theta = \theta \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}, \tag{1.1}$$

where $G_{\mu\nu}$ is the gluonic field strength, $\tilde{G}^{\mu\nu}$ is its dual and α_s is the strong coupling constant. The $G_{\mu\nu}\tilde{G}^{\mu\nu}$ quantity is a total derivative, consequently it can contribute to the physical observables only through non perturbative effects. The NEDM is related to the $\bar{\theta}$-angle by the following relation :

$$d_n \sim \frac{e}{M_n}(\frac{m_q}{M_n})\bar{\theta} \sim \{ \begin{matrix} 2.7 \times 10^{-16}\overline{\theta} & [5] \\ 5.2 \times 10^{-16}\overline{\theta} & [6] \end{matrix} \tag{1.2}$$

and consequently, according to the experimental measurements $d_n < 1.1 \times 10^{-25}ecm$ [7], the $\bar{\theta}$ parameter must be less than 2×10^{-10} [8]. The well known strong CP problem consists in explaining the smallness of $\bar{\theta}$. In this regard, several scenarios were suggested. The most known one was proposed by Peccei and Quinn [10] and consists in implementing an extra $U_A(1)$ symmetry which permits a dynamical suppression of the undesired θ-term. This is possible

due to the fact that the axial current J_5^μ is related to the gluonic field strength through the following relation $\partial_\mu J_5^\mu = \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$. The breakdown of the $U_A(1)$ symmetry gives arise to a very light pseudogoldstone boson called axion. This particle may well be important to the puzzle of dark matter and might constitute the missing mass of the universe [11].

Motivated by: (a) the direct relation between the $\bar{\theta}$-angle and NEDM d_n, as it was demonstrated firstly in [6] via the chiral perturbation theory and recently in [12, 13] within QCD sum rules formalism; (b) the possibility to restore some broken symmetries by increasing the temperature; we shall use the QCD sum rules at $T \neq 0$ [20] to derive thermal dependence of the ratio $|\frac{d_n}{\theta}|$. Then we study its thermal behaviour at low temperatures and discuss the consequences of temperature effects on the restoration of the broken CP symmetry.

This paper is organized as follows: Section 2 is devoted to the calculations of the NEDM induced by the $\bar{\theta}$ parameter from QCD sum rules. In section 3, we show how one introduces temperature in QCD sum rules calculations. We end this paper with a discussion and qualitative analysis of the thermal effects on the CP symmetry.

2   NEDM from QCD sum rules

In the two later decades, QCD sum rules à la SVZ [14] were applied successfully to the investigation of hadronic properties at low energies. In order to derive the NEDM through this approach, many calculations were performed in the literature [15, 29]. One of them, which turns out to be more practical for our study, has been obtained recently in [12, 13]. It consists in considering a lagrangian containing the following P and CP violating operators:

$$L_{P,CP} = -\theta_q m_* \sum_f \bar{q}_f i\gamma_5 q_f + \theta\frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}. \tag{2.1}$$

θ_q and θ are respectively two angles coming from the chiral and the topological terms and m_* is the quark reduced mass given by $m_* = \frac{m_u m_d}{m_u + m_d}$. The authors of [13] start from the two points correlation function in QCD background with a nonvanishing θ and in the presence of a constant external electomagnetic field $F^{\mu\nu}$:

$$\Pi(q^2) = i \int d^4x e^{iqx} < 0|T\{\eta(x)\bar{\eta}(0)\}|0 >_{\theta,F} . \tag{2.2}$$

$\eta(x)$ is the interpolating current which in the case of the neutron reads as [16]:

$$\eta = 2\epsilon_{abc}\{(d_a^T C\gamma_5 u_b)d_c + \beta(d_a^T C u_b)\gamma_5 d_c\}, \tag{2.3}$$

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where β is a mixing parameter. Using the operator product expansion (OPE), they have first performed the calculation of $\Pi(q^2)$ as a function of matrix elements and Wilson coefficients and then have confronted the QCD expression of $\Pi(q^2)$ to its phenomenological parametrisation. $\Pi(q^2)$ can be expanded in terms of the electromagnetic charge as[15]:

$$\Pi(q^2) = \Pi^{(0)}(q^2) + e\Pi^{(1)}(q^2, F^{\mu\nu}) + O(e^2). \tag{2.4}$$

The first term $\Pi^{(0)}(q^2)$ is the nucleon propagator which include only the CP-even parameters [19, 18], while the second term $\Pi^{(1)}(q^2, F^{\mu\nu})$ is the polarization tensor which may be expanded through Wilson OPE as: $\sum C_n < 0|\bar{q}\Gamma q|0 >_{\theta,F}$, where Γ is an arbitrary Lorentz structure and C_n are the Wilson coefficient functions calculable in perturbation theory [19]. From this expansion, one keeps only the CP-odd contribution piece. By considering the anomalous axial current, one obtains the following θ dependence of $< 0|\bar{q}\Gamma q|0 >_\theta$ matrix elements [12]:

$$m_q < 0|\bar{q}\Gamma q|0 >_\theta = im_*\theta < 0|\bar{q}\Gamma q|0 >, \tag{2.5}$$

where m_q and m_* are respectively the quark and reduced masses. The electromagnetic dependence of these matrix elements can be parametrized through the implementation of the κ, χ and ξ susceptibilities defined as [18]:

$$\begin{aligned} < 0|\bar{q}\sigma^{\mu\nu}q|0 >_F &= \chi F^{\mu\nu} < 0|\bar{q}q|0 > \\ g < 0|\bar{q}G^{\mu\nu}q|0 >_F &= \kappa F^{\mu\nu} < 0|\bar{q}q|0 > \\ 2g < 0|\bar{q}\tilde{G}^{\mu\nu}q|0 >_F &= \xi F^{\mu\nu} < 0|\bar{q}q|0 > . \end{aligned} \tag{2.6}$$

Putting altogether the above ingredients and after a straightforward calculation [13], the following expression of $\Pi^{(1)}(q^2, F^{\mu\nu})$ for the neutron is derived:

$$\begin{aligned} \Pi(-q^2) = &-\frac{\bar{\theta}m_*}{64\pi^2} < 0|\bar{q}q|0 > \{\tilde{F}\sigma, \hat{q}\}[\chi(\beta+1)^2(4e_d - e_u)\ln(\frac{\Lambda^2}{-q^2}) \\ &-4(\beta-1)^2 e_d(1 + \frac{1}{4}(2\kappa + \xi))(\ln(\frac{-q^2}{\mu_{IR}^2}) - 1)\frac{1}{-q^2} \\ &-\frac{\xi}{2}((4\beta^2 - 4\beta + 2)e_d + (3\beta^2 + 2\beta + 1)e_u)\frac{1}{-q^2}...], \end{aligned} \tag{2.7}$$

where $\bar{\theta} = \theta + \theta_q$ is the physical phase and $\hat{q} = q_\mu\gamma^\mu$.

The QCD expression (2.7) will be confronted to the phenomenological parametrisation $\Pi^{Phen}(-q^2)$ written in terms of the Neutron hadronic properties. The latter is given by:

$$\Pi^{Phen}(-q^2) = \{\tilde{F}\sigma, \hat{q}\}(\frac{\lambda^2 d_n m_n}{(q^2 - m_n^2)^2} + \frac{A}{(q^2 - m_n^2)} + ...), \tag{2.8}$$

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where m_n is the neutron mass, e_q is the quark charge. A and λ^2, which originate from the phenomenological side of the sum rule, represent respectively a constant of dimension 2 and the neutron coupling constant to the interpolating current $\eta(x)$. This coupling is defined via a spinor v as $<0|\eta(x)|n>=\lambda v$.

3   QCD sum rules at finite temperature

The introduction of finite temperature effects may provide more precision to the phenomenological values of hadronic observables. Within the framework of QCD sum rules, the T-evolution of the correlation functions appear as a thermal average of the local operators in the Wilson expansion[20, 21, 24]. Hence, at nonzero temperature and in the approximation of the non interacting gas of bosons (pions), the vacuum condensates can be written as :

$$< O^i >_T = < O^i > + \int \frac{d^3p}{2\epsilon(2\pi)^3} < \pi(p)|O^i|\pi(p) > n_B(\frac{\epsilon}{T}) \tag{3.1}$$

where $\epsilon = \sqrt{p^2 + m_\pi^2}$, $n_B = \frac{1}{e^x - 1}$ is the Bose-Einstein distribution and $< O^i >$ is the standard vacuum condensate (i.e. at T=0). In the low temperature region, the effects of heavier resonances ($\Gamma = K, \eta, ..etc$) can be neglected due to their distibution functions $\sim e^{\frac{-m_\Gamma}{T}}$[23]. To compute the pion matrix elements, we apply the soft pion theorem given by:

$$< \pi(p)|O^i|\pi(p) > = -\frac{1}{f_\pi^2} < 0|[F_5^a, [F_5^a, O^i]]|0 > +O(\frac{m_\pi^2}{\Lambda^2}), \tag{3.2}$$

where Λ is a hadron scale and F_5^a is the isovector axial charge:

$$F_5^a = \int d^3x \bar{q}(x)\gamma_0\gamma_5\frac{\tau^a}{2}q(x). \tag{3.3}$$

Direct application of the above formula to the quark and gluon condensates shows that [22, 23]: (i) Only $< \bar{q}q >$ is sensitive to temperature. Its behaviour at finite T is given by:

$$< \bar{q}q >_T \simeq (1 - \frac{\varphi(T)}{8}) < \bar{q}q >, \tag{3.4}$$

where $\varphi(T) = \frac{T^2}{f_\pi^2}B(\frac{m_\pi}{T})$ with $B(z) = \frac{6}{\pi^2}\int_z^\infty dy\frac{\sqrt{y^2-z^2}}{e^y-1}$ and f_π is the pion decay constant ($f_\pi \simeq 93 MeV$). The variation with temperature of the quark condensate $< \bar{q}q >_T$ results in two different asymptotic behaviours, namely:
$< \bar{q}q >_T \simeq (1-\frac{T^2}{8f_\pi^2}) < \bar{q}q >$ for $\frac{m_\pi}{T} \ll 1$, and $< \bar{q}q >_T \simeq (1-\frac{T^2}{8f_\pi^2}e^{\frac{-m_\pi}{T}}) < \bar{q}q >$ for $\frac{m_\pi}{T} \gg 1$.

(ii) The gluon condensate is nearly constant at low temperature and a T dependence occurs only at order T^8.

As usual, the determination of the ratio $\frac{d_n}{\bar{\theta}}$ sum rules at non zero temperature is now easily performed through two steps. In the first step, we apply Borel operator to both expressions of the Neutron correlation function shown in Eqs. (2.7) and (2.8), where finite temperature effects were introduced as discussed above. Next step, by invoking the quark-hadron duality principle, we deduce the following relation of the $\bar{\theta}$ induced NEDM:

$$\frac{d_n}{\bar{\theta}}(T) = -\frac{M^2 m_*}{16\pi^2} \frac{1}{\lambda_n^2(T) M_n(T)} (1 - \frac{\varphi(T)}{8}) < \bar{q}q > [4\chi(4e_u - e_d) - \frac{\xi}{2M^2}(4e_u + 8e_d)]e^{\frac{M_n^2}{M^2}}, \quad (3.5)$$

where M represents the Borel parameter. Note that we have neglected the single pole contribution entering via the constant A, as suggested in [12].

The expression (3.5) is derived with $\beta = 1$ which is more appropriate for us since it suppresses the infrared divergences. In fact, the Ioffe choice ($\beta = -1$) which is rather more useful for the CP even case, removes the leading order contribution in the sum rules (2.7). The coupling constant $\lambda_n^2(T)$ and the neutron mass $M_n(T)$ which appear in (3.5) were determined from the thermal QCD sum rules. For the former, we consider the $\hat{q}$ sum rules in [23, 25] with $\beta = 1$ and then we extract the following explicit expression of $\lambda_n^2(T)$:

$$\lambda_n^2(T) = \{\frac{3}{8(2\pi)^4}M^6 + \frac{3}{16(2\pi)^2}M^2 < \frac{\alpha_s}{\pi}G^2 >\}\{1 - (1 + \frac{g_{\pi NN}^2 f_\pi^2}{M_n^2})\frac{\varphi(T)}{16}\}e^{\frac{M_n^2(0)}{M^2}} \quad (3.6)$$

Within the pion gas approximation, Eletsky has demonstrated in [26] that inclusion of the contribution coming from the pion-nucleon scattering in the nucleon sum rules is mandatory. The latter enters Eq.(3.6) through the coupling constant $g_{\pi NN}$, whose values lie within the range 13.5-14.3 [27].

Numerical analysis is performed with the following input parameters: the Borel mass has been chosen within the values $M^2 = 0.55 - 0.7 GeV^2$ which correspond to the optimal range (Borel window) in the $\frac{d_n}{\bar{\theta}}$ sum rule at $T = 0$ [13]. For the χ and ξ susceptibilities we take $\chi = -5.7 \pm 0.6 GeV^{-2}$ [28] and $\xi = -0.74 \pm 0.2$ [29]. As to the vacuum condensates appearing in (3.5), we fix $< \bar{q}q >$ and $< G^2 >$ to their standard values [14].

4   Discussion and Conclusion

In the two above sections, we have established the relation between the NEDM and $\bar{\theta}$ angle at non zero temperature from QCD sum rules. Since the ratio $\frac{d_n}{\bar{\theta}}$ is expressed in terms

of the pion parameters f_π, m_π and of $g_{\pi NN}$, we briefly recall the main features of their thermal behaviour. Various studies performed either within the framework of the chiral perturbation theory and/or QCD sum rules at low temperature have shown the following features:

(i) The existence of QCD phase transition temperature T_c which signals both QCD deconfinement and chiral symmetry restoration [20, 31].

(ii) f_π and $g_{\pi NN}$ have very small variation with temperature up to T_c. So, we shall assume them as constants below T_c. However, they vanish if the temperature passes through the critical value T_c [30].

(iii) The thermal mass shift of the neutron and the pion is absent at order $O(T^2)$ [23, 21]. δM_n shows up only at the next order T^4, but its value is negligible [26].

By taking into account the above properties, we plot the ratio defined in Eq. (3.5) as a function of T. From the figure, we learn that the ratio $\mid \frac{d_n}{\theta} \mid$ survives at finite temperature and it decreases smoothly with T (about 16% variation for temperature values up to 200 MeV). This means that either the NEDM value decreases or $\bar{\theta}$ increases. Consequently, for a fixed value of $\bar{\theta}$ the NEDM decreases but it does not exhibit any critical behaviour. Furthermore, if we start from a non vanishing $\bar{\theta}$ value at $T = 0$, it is not possible to remove it at finite temperature. We also note that $\mid \frac{d_n}{\theta} \mid$ grows as M^2 or χ susceptibility increases. It also grows with quark condensate rising. However this ratio is insensitive to both the ξ susceptibility and the coupling constant $g_{\pi NN}$. We notice that for higher temperatures, the curve $\mid \frac{d_n}{\theta} \mid = f(\frac{T}{T_c})$ exhibits a brutal increase justified by the fact that for temperatures beyond the critical value T_c, at which the chiral symmetry is restored, the constants f_π and $g_{\pi NN}$ become zero and consequently from Eq(3.5) the ratio $\frac{d_n}{\theta}$ behaves as a non vanishing constant. The large difference between the values of the ratio for $T < T_c$ and $T > T_c$ maybe a consequence of the fact that we have neglected other contributions to the the spectral function, like the scattering process $N + \pi \rightarrow \Delta$. These contributions , which are of the order T^4, are negligible in the low temperture region but become substantial for $T \geq T_c$. Moreover, this difference may also originate from the use of soft pion approximation which is valid essentially for low T ($T < T_c$). Therefore it is clear from this qualitative analysis, which is based on the soft pion approximation, that the temperature does not play a fundamental role in the suppression of the undesired θ-term and hence the broken CP symmetry is not restored, as expected. This is not strange, in fact it was shown that more heat does not imply automatically more symmetry [33, 36]. Moreover, some exact symmetries can be broken by increasing temperature [32, 33]. The symmetry non restoration phenomenon, which means that a broken symmetry at T=0 remains broken even at high temperature, is essential for discrete symmetries, CP symmetry

in particular. Indeed, the symmetry non restoration allows us to avoid wall domains inherited after the phase transition [34] and to explain the baryogenesis phenomenon in cosmology [35]. Furthermore, it can be very useful for solving the monopole problem in grand unified theories [36].

Acknowledgments

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Figure Captions

Figure: Temperature dependence of the ratio $\mid \frac{d_n}{\theta} \mid$